CALCULATION OF THE MOTION OF A GAS BEHIND THE FRONT OF A LUMINOUS DETONATION WAVE TAKING ACCOUNT OF THE LATERAL EXPANSION OF THE PLASMA COLUMN

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Nonsteady-state gasdynamic processes are considered in a plasma column (hot channel), formed behind the front of a shock wave moving toward a laser beam. A quasi-one-dimensional approximation is used – the parameters in the channel are assumed to be compensated with respect to cross section, but depend on the time and distance along the axis. Motion in the cold dense casing surrounding the channel is assumed to be one-dimensional and cylindrically symmetrical. The solutions of the corresponding systems of equations in partial derivatives permit the parameters to be determined approximately both in the case when the mean free path of the radiation is small in comparison with the radius of the beam (luminous detonation) and also in the case when the mean free path is comparable with the radius. Examples are given of the corresponding numerical calculations. It is shown that in the latter case, a cycle of incomplete absorption can be achieved when behind the shock wave front, moving with constant velocity up to the Jouguet plane, only a part of the radiation energy incident on the front is released.

§ 1. The concept of luminous detonation – the propagation of a shock wave counter to a laser beam, behind the front of which the radiation energy which defines the parameters of the wave is released in a narrow zone – was used for the first time in [1] in order to interpret the phenomenon of the propagation of a plasma front from a laser burst. In order to calculate the parameters of the detonation wave, ordinary algebraic relations are used: the hydrodynamic laws of conservation on the assumption of total absorption in a narrow zone of the radiation incident on the front and the fulfilling of the Jouguet condition [1-4]. However, these assumptions are not always fulfilled. Moreover, the parameters of the plasma behind the detonation wave front, moving with variable velocity, are interesting. In these instances, it is natural to have recourse to the solution of the corresponding gasdynamic problem, taking account of the finiteness of the energy release zone. When the diameter of the beam is larger in comparison with the distance traveled by the shock wave, the pattern is close to plane.

Sometimes it is possible to use also self-similar solutions (see, for example, [5]), and in the general case recourse can be had to any numerical method (for example, the one used in [6]) for calculating the occurrence and propagation of detonation waves in a layer of the vapors formed by the action of radiation on condensed substances. However, since the diameter of the beam usually is extremely small, during the action of the radiation pulse the shock wave travels distances which are greater than the diameter, and the spreading-out of the plasma column, formed behind the shock front in the direction perpendicular to the beam, becomes considerable. Expansion and cooling of the plasma can lead to an increase of its transparency, a reduction of the energy release behind the shock front, and to the impossibility of detonation when the diameter of the beam is less than a certain critical value [7].

The solution of the complete two-dimensional non-steady-state gasdynamic problem with the release of energy is quite complex and time consuming, and numerous alternatives are possible, which differ in the density of the radiation flux, the duration of action, beam dimensions, the shape of the pulse with time, and the density and composition of the gas, according to which the shock waves are propagated. Therefore, it is

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advantageous to develop an approximate method for calculating the parameters, taking into account the effect of lateral expansion.

The phenomenon being considered is characterized, in the first place, by high sonic velocities in the hot "channel" in which the gas has been heated up not only by the shock front moving counter to the radiation, but also as a result of the absorption of radiation behind the front; secondly, by relatively low velocities of sound in the cold "envelope," where the gas is compressed behind the "lateral" shock front, which has, in addition, a lower amplitude than the "head" wave. In order to describe the motion of the gas and the change of parameters in a long narrow channel, we shall use a quasi-one-dimensional approximation – the parameters in each section of the "channel" are assumed to be compensated, but their change along the axis of the channel and with time is postulated.

The motion of the gas in the envelope will be assumed to be one-dimensional and traveling in the direction perpendicular to the axis of the channel. In order to determine the parameters in the envelope, it is necessary to solve the cylindrically symmetrical problem for each section (realistic, of course, for a number of chosen sections). The parameters in each section are related with one another by the parameters in the channel, where the energy of the particles is changing both on account of its external supply and its redistribution during motion along the axis, and also due to expansion of the channel. The quasi-one-dimensional approximation is applicable also in another limiting case – when the pattern is close to plane and the lateral expansion is negligibly small.

The system of equations for describing the gasdynamic processes in the channel, taking account of the change of its cross-sectional area, has the form

$$\partial u/\partial t + S\partial p/\partial m = 0;\ \partial\varphi/\partial t - \partial u/\partial m = 0; \tag{1.1}$$

$$\partial E/\partial t + \partial(puS)/\partial m + \partial F/\partial m = f - 2p\varphi w_0 r_0; \tag{1.2}$$

$$\partial S/\partial t - (u/\varphi)\partial S/\partial m = 2w_0 r_0; \tag{1.3}$$

$$E = e + u^2/2,\quad S = r_0^2,\quad \varphi^{-1} = \rho S, \tag{1.4}$$

where p is the pressure; ρ is the density; e is the specific energy of unit mass; u is the velocity; E is the total energy of the particles; πS is the cross-sectional area of the channel; r_0 is the radius of the channel; φ is a parameter, which is the analog of the specific volume; w_0 is the velocity of motion of the boundaries of the channel (in the direction perpendicular to the axis); t is the time; and m is the Lagrangian mass coordinate. The system of equations of motion and continuity (1.1), energy (1.2), and the equation for determining the area of the channel (1.3) must be added to the equation of state

$$p = (\gamma - 1)e\rho,\ \gamma = \gamma(e,\ \rho), \tag{1.5}$$

where γ is the effective integrated adiabatic index. The distance x along the axis of the channel can be determined, for example, from the relation

$$\rho S \partial x/dm = 1.$$

Equation (1.3) is obtained by converting to m and t from the relation

$$\partial r_0/\partial t|_{x=\mathrm{const}} = w_0.$$

The last term of Eq. (1.2) describes the work completed during expansion of the channel, and the penultimate term describes the intensity of the heat release ($f > 0$), for example, Joule heat, during the passage of an electric current, or the removal of heat ($f < 0$) through the wall of the channel due to electron thermal conductivity or by means of spatial luminescence. Below, we shall suppose that these losses of energy are negligibly small, and therefore we shall not specify the connection between f and the other gas parameters, assuming simply that $f = 0$.

The quantity F is the total energy flux along the channel. The corresponding energy release $\partial F/\partial m$ is assumed to be identical for all particles of given cross section.

In this paper, we shall assume that the transfer of energy along the channel is accomplished only because of the absorption of monochromatic laser radiation. In this case, we shall use the transport equation

$$\partial F/\partial m = -(\varkappa/S)F \quad \text{or} \quad \partial F/\partial x = -\eta F,$$

averaged over the cross section, where η is the linear and $\varkappa$ the mass absorption coefficient, and $\varkappa = \varkappa(e, \rho)$. The diffusion of the flow of radiation energy over the whole cross section of the channel contributes a finite error in the description of the picture of the process. However, if the energy release takes place only in a relatively narrow zone, within the limits of which the cross sectional area S could not be changed by comparison with its initial value, equal to the cross section of the beam, as occurs, for example, at the detonation wave front, then this procedure does not contribute a significant error.

The system of equations for describing the motion in the envelope between the shock wave and the channel has the form

$$\partial w/\partial t + r\partial p/\partial u = 0;\ \partial v/\partial t - \partial(wr)/\partial n = 0; \tag{1.6}$$

$$[\partial(e + w^2/2)]/\partial t + \partial(pwr)/\partial n = f, \tag{1.7}$$

where v is the specific volume of the gas $(v = 1/\rho)$; w is the velocity of motion in the direction perpendicular to the axis; and n is the Lagrangian mass coordinate, connected with the radius r by the relation

$$\rho r\partial r/\partial n = 1. \tag{1.8}$$

The quantity f in Eq. (1.7), determines the energy release in the envelope, for example, due to absorption of radiation of the continuum, emitted by the hot channel. In the specific examples considered below, we shall assume that $f = 0$. We shall call the system of equations (1.1)-(1.5), system I, and system of equations (1.5), (1.6)-(1.8), system II.

System II will be solved for each of the sections considered $x = x_i$ with the boundary condition defining equality of pressure in the cold envelope and in the hot channel at their boundary of separation:

$$p_0(t) = p(t, n = 0) = p(t, m_i(t)).$$

where $m_i(t)$ corresponds to a given value of x_i. At the same time, the densities and temperatures in the channel and envelope are different. We note that particles in the channel are transported relative to a given section and the presence of this contact discontinuity would make it difficult to carry out two-dimensional calculations of this problem by a direct scheme. Obviously, in this case, it is desirable to separate the boundaries of separation of the channel and the envelope.

The values of the velocity at the boundary $w_0 = w(t, n = 0)$ are used in the solution of system I jointly with system II. In order to reduce the number of sections x_i for which the calculation of system II will be carried out, it is necessary to choose a suitable method of interpolation of the parameters between these sections. We introduce

$$\alpha(x, t) = w_0/w_s,\ \beta(x, t) = p_0/p_s,$$

where w_s and p_s are the velocity and pressure at the shock front, related to each other by the Hugoniot adiabat

$$p_s = p_H(w_s). \tag{1.9}$$

The quantities α and β are ordinary weakly varying functions of their arguments. For a strong shock wave, moving according to a power law from a piston, expanding according to this same law, the problem concerning the motion of the gas between them is self-similar and the quantities α and β are constant in time. This occurs also for the limiting case of a cylindrical powerful explosion with constant energy. Therefore, relation (1.9) can be treated as the relation between p_0 and w_0, and it can be used as the boundary condition for solving system I:

$$p_0 = \beta p_H(w_0/\alpha).$$

In this case, the values of $\alpha(x, t)$ and $\beta(x, t)$ in a certain time interval are assumed to be equal to their value in the last instant of the preceding interval, i.e., their extrapolation is used. Later [when system II is solved with the law of pressure variation $p_0(t)$ obtained in the solution of system I], a recalculation can be carried out and the refined functions $\alpha(x_i, t)$ and $\beta(x_i, t)$ can be determined at a given time interval. For the lengths between the sections in which system II was solved, the functions α and β are determined by interpolation with respect to x.

For calculations which do not require special accuracy, in those cases when the "lateral" shock wave is strong and the compression behind its front is large, the approximation of an infinitely thin envelope can be used when we have a simple relation between α, β, and the law of motion of the envelope [8, 9]

$$\alpha = 1,\ \beta = 1 + d\ln w_0/d\ln n_s,$$

where n_s is the mass entrained by the lateral shock front.

Numerical calculations of the flow in the channel have been carried out by the difference scheme described in [10] using an artificial viscosity and based on the method of integral relations. Calculations of the motion in a lateral direction have been carried out by the scheme with a formulated shock front, similar to [11], or on the assumption of an infinitely thin envelope. We note that the use of the latter assumption even for a strong wave may prove to be extremely rough. If the hot channel is open from one or both sides, and flow into a vacuum or into a low-density medium takes place from it, then the pressure in the channel falls sharply, as a result of which the return motion of the walls of the channel toward its axis commences, while the shock front will be moving from the center, i.e., the quantity β can be reduced sharply and may even change sign. In this situation it becomes necessary to carry out the calculation of the motion in the envelope.

The cases of a closed channel, in which no reverse motion occurs, will be considered in this paper. The calculations of the problem of cylindrically symmetrical motion of a gas, with the laws of fall of pressure in the channel with time, and which are typical for the problems considered below, have shown that over a quite large time interval $\alpha \approx 1$ and $\beta=0.7$. These constant values for α and β were assumed in the examples of calculations described.

When using this method, it is necessary also that the shape of the channel and of the shock wave do not differ too strongly from a circular cylinder, in order that only the redistribution of energy between sections due to motion in the channel would be taken into account, and not redistribution due to motion in the cold envelope in the direction along the axis of the channel.

We note that an approach close to that described was used in an investigation of the dynamics of constriction of a plasma pinch by an electric discharge in vacuo in [12], where there was no cold gas envelope, and the pressure of the magnetic field was determined by the magnitude of the electric current flowing and the radius of the channel. The difference between our formulation and that of [12] consists in that the total energy equation for particles in the channel is considered, and it is not postulated that the flow is adiabatic or isothermal. Thus, the formation and propagation of shock waves in the channel is permitted.

In [13] an even more simplified approach was used for describing the parameters behind the front of a luminous detonation (the motion was assumed to be cylindrically symmetrical). It is obvious that with a strong change of the radiation flux density with time and with a corresponding change of velocity of the luminous detonation front and the gas velocity behind it, a significant motion of the plasma in the channel along its axis originates. Moreover, it will be shown below that even with a constant flux density, in consequence of the strong pressure differential between regions close to the front of the detonation wave, where the pressure is maintained continuously at a high level, and remote from it where the pressure is reduced strongly because of the lateral expansion of the channel, motion along the channel axis originates which is directed from the luminous detonation front. Finally, the quasi-one-dimensional procedure enables the parameters to be calculated also by taking account of the finiteness of the width of the energy release zone, however, with a not too large magnitude of the radiation mean free path at the shock front.

Let us consider an example of a calculation of the problem of a gas behind the front of a luminous detonation wave in air at normal density, $\rho_a = \rho_L = 1.29 \cdot 10^{-3}$ g/cm^3, with a flux density of the incident radiation $q_a = 250$ MW/cm^2 and a radius R = 3 mm. The thermodynamic properties of the heated air were described in the detailed tables of [14] and the optical properties, in the tables of [15]. We note that in this example the magnitudes of the radiation mean free paths behind the wave front are found to be very small in comparison with the radius of the channel, and therefore in the calculations we used a method of artificial limitation of the magnitude of the absorption coefficient in order to spread out the energy release zone at several calculated points [6].

Figure 1 shows the shape of the channel. The numbers above the curves correspond to time instants (μsec) from the start of the effect of the laser emission on the initially formed plasma layer of small thickness. The detonation initiation process and the formation process of this layer will not be considered here. The initial temperature and density of the plasma in this thin layer were chosen approximately with the corresponding values in the vapor of a solid substance at the instant of initiation of the "burst" in them [6].

At the instant t = 1.14 μsec, the radiation ceases to act. After this time, energy E = 285 J/cm^2 is supplied to unit area and with a spot area of S = 0.283 cm^2 the total radiation energy is ES = 80 J. The channel at this instant of time is quite long and narrow with a slightly changing (both in time and in length) angle of inclination of its walls in relation to the axis.

We note that the ratio the channel length to its radius of 2 : 1 is maintained for a long time after the laser is switched off – right up to instants of about 3 to 4 μsec. Further calculation was terminated, as the shock

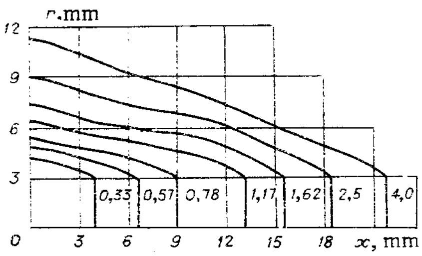

Fig. 1

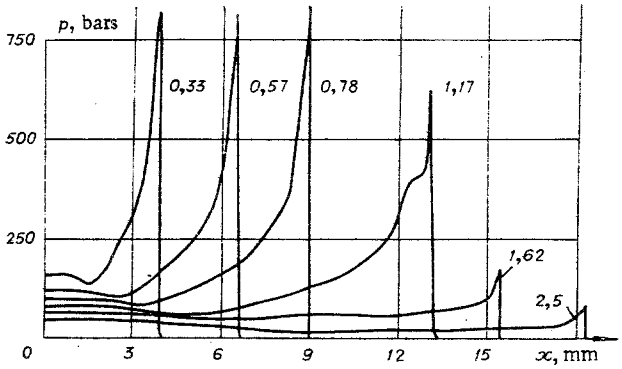

Fig. 2

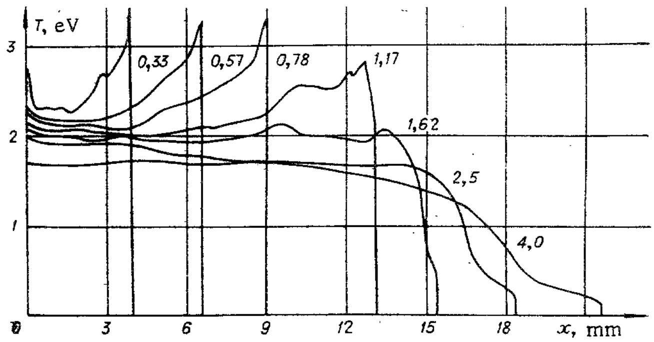

Fig. 3

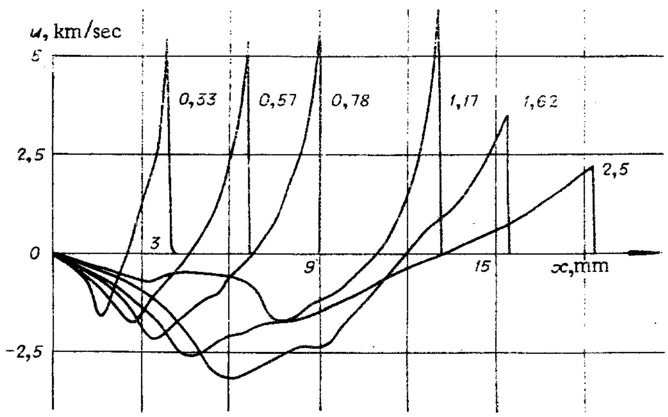

Fig. 4

wave ceases to be strong (the pressure in it becomes equal to 15-20 atm approximately). Figure 2 represents the pressure distribution along the axis of the channel at different instants. It can be seen that the pressure in the vicinity of the solid surface from which the detonation wave is moving is considerably lower than the pressure at the shock front, and after the instant of switching-off the radiation source a sharp reduction of the amplitude of the front begins. Figure 3 shows the temperature distribution. The temperature drop along the axis of the channel is small right up to the instant of switching-off the source, when very cold layers appear in the vicinity of the front of the rapidly attenuating shock wave.

The small increase of temperature and pressure in the vicinity of the obstacle from which the detonation wave is moving is associated with retardation of the jet of gas, flowing as from a nozzle from the shock front. This is clearly seen in Fig. 4, where the velocity distribution along the axis of the channel is shown.

Figures 2 to 4 do not show the narrow zone between the shock front and the Jouguet plane. In this case, the parameters at the shock front are as follows: pressure $p_s = 1710$ bars, temperature $T_s = 20{,}600°K$, and the compression $Q_s = 11.1$.

§ 2. In the version considered, the magnitudes of the radiation mean free paths in the Jouguet plane were small in comparison with the radius of the channel. During the reduction of the radiation flux density to approximately 80 MW/cm^2, when the temperature at the shock front amounts to approximately 1 eV (in the Jouguet plane it is approximately 2 eV), the magnitudes of the mean free paths of the radiation from a neodymium laser increase correspondingly up to approximately 2 mm and the detonation collapses. With a reduction of the air density for a fixed flux density, the wave velocities and the temperature increase, which leads to a reduction of the detonation limits.

At a density equal to 1/10 of normal, this same magnitude of the mean free path occurs only at 13 MW/cm^2. With a reduction of the air density, the minimum radiation mean free path is increased and, finally, a situation occurs when the finiteness of the energy release zone must be taken into account for any flux density and a luminous detonation in the usual sense generally is not possible. However, cycles becomes possible which are similar to luminous detonation with the partial absorption of the energy of the incident radiation.

Let us obtain the relations for determining the parameters of the gas behind the fronts of a luminous detonation, taking into account the variability of the adiabatic index $\gamma = \gamma(e, \rho)$, since in [1-4] they were derived only for the case of $\gamma = \text{const}$. We shall distinguish the integral adiabatic exponent occurring in the equation of state (1.5) and the differential adiabatic exponent $k(e, \rho)$ which occurs in the determination of the velocity of sound c,

$$c^2 = kp/\rho,\ k = \gamma + 1/(\gamma - 1) \cdot \partial\gamma/\partial \ln \rho\,|_e + \partial\gamma/\partial \ln e\,|_\rho. \tag{2.1}$$

We shall mark all the parameters ahead of the detonation wave with the subscript a, at the shock front by the subscript s, and in the Jouguet plane by the subscript j. We shall assume the detonation wave to be strong, and therefore we shall put $u_a = p_a = \rho_a = 0$. We shall use the conventional laws of conservation of mass, momentum, and energy

$$\begin{gathered} -Mu_j + p_j = -Mu_s + p_s = 0; \\ Mv_j + u_j = Mv_s + u_s = Mv_a, \\ -M\left(e_j + u_j^2/2\right) + p_j u_j + q_j = -M\left(e_s + u_s^2/2\right) + p_s u_s + q_s = q_a, \end{gathered} \tag{2.2}$$

where q is the radiation flux density; M is the mass flow through unit area of the front. Neglecting radiation absorption ahead of the shock front (in the "heated-up" zone), i.e., assuming $q_s = q_a$, we add to Eq. (2.2) the Jouguet condition

$$M = \rho_a D = \rho_j c_j,$$

where D is the detonation velocity; c_j is determined by Eq. (2.1) when $\rho = \rho_j$ and $e = e_j$. Using the equation of state (1.5), we obtain the following relations for determining the parameters in the Jouguet plane:

$$\begin{gathered} q_e = q_a - q_j = -[\rho_a D^3(2k_j + 1 - \gamma_j)]/[2(k_j + 1)^2(\gamma_j - 1)]; \\ e_j = k_j D^2/[(k_j + 1)^2(\gamma_j - 1)];\ u_j = D/(k_j + 1); \\ \theta_j = \rho_j/\rho_a = (k_j + 1)/k_j;\ \rho_j = \rho_a D^2/(k_j + 1), \end{gathered}$$

where θ_j is the compression of the gas in the Jouguet plane; q_e is the effective radiation flux density, corresponding to the energy absorbed in the front. In the case of total radiation absorption, i.e., when $q_j = 0$, the quantity q_e coincides with q_a, the radiation flux density incident on the front. We note that the sign of D and M is

TABLE 1

q, MW/cm²	D, km/sec	T_s, K	T_j, K	η_s, cm^{-1}	η_j, cm^{-1}
3	7.74	8 670	17 900	0.01	1.1
4	8.57	9 480	19 800	0.03	1.3
5	9.34	10 600	21 700	0.14	0.70
7	10.9	13 800	26 500	2.1	0.65
10	12.5	17 000	33 000	7.6	1.0
15	14.2	19 600	38 300	15	1.7
20	15.6	21 700	42 300	12	2.3
25	16.8	23 700	46 100	15	2.7
30	18.0	25 700	50 000	18	3.1
40	19.9	30 500	56 900	18	3.9
50	21.4	34 500	62 400	20	4.7
70	24.0	40 700	71 100	30	6.1
100	27.1	46 800	82 400	52	7.8
150	31.1	55 100	97 400	74	11
200	34.4	62 700	110 000	89	13
250	37.2	69 300	121 000	110	15
300	39.8	75 200	132 000	120	16

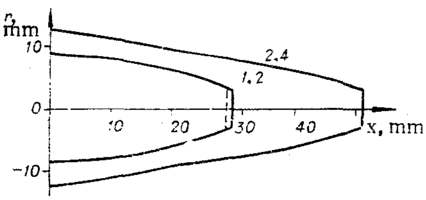

Fig. 5

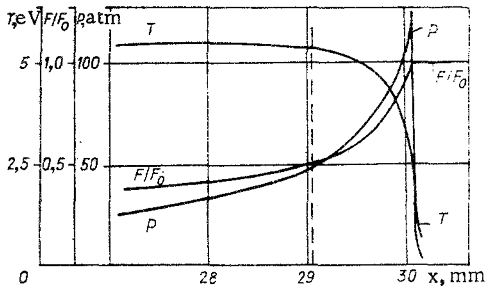

Fig. 6

contrary to the sign of q_e and q_a – the detonation wave is moving toward the radiation. The parameters at the shock front are determined in the usual way:

$$e_s = u_s^2/2; \quad u_s = [2/(\gamma_s + 1)]D;$$
$$\theta_s = \rho_s/\rho_a = (\gamma_s + 1)/(\gamma_s - 1); \; p_s = 2\rho_a D^2/(\gamma_s + 1).$$

Here θ_s is the compression of the gas at the shock front. The quantities γ_j, k_j and k_s, γ_s depend on e_j, ρ_j and e_s, ρ_s, respectively. Therefore, the calculation of the algebraic systems is carried out by the method of iterations.

The results of the calculations of the detonation wave parameters for a density equal to 0.0316 of the normal are given in Table 1. Here η_s and η_j are the linear absorption coefficients in the plane of the front and in the Jouguet plane for an energy of the quanta $\varepsilon = 1.16$ eV (neodymium laser).

Analyzing the data given in the table, we can satisfy ourselves that at low radiation flux densities detonation is impossible even at high beam radii, and this is associated with the high transmittance of the air behind the shock front. In the region of low temperatures, the quantity η depends sharply on T, and $\eta_s \ll \eta_j$. Consequently, behind the shock front there should exist an extended zone (with thickness of order $l_s = \eta_s^{-1}$, behind which should be an abruptly heated front $l_j = \eta_j^{-1}$). If $\eta_s R \lessdot 1$, then the lateral relief behind the shock front leads to cooling of the gas and to a considerable reduction of η, i.e., heating-up and the onset of strong absorption become impossible. A different situation occurs in the region of high temperatures. This is associated with the fact that with the completion of the single and initial multiple ionization, when the magnitude of the average charge increases with temperature, the absorption coefficient also increases with temperature but to a lesser degree. The density at the shock front is significantly higher than in the Jouguet plane (the compression $\theta_s = 13.7$ and 10.8, while $\theta_j = 1.83$ and 1.78 at the beginning and end of the table). Because of this, η_s is markedly greater than η_j, and, consequently, the energy release zone is extended markedly according to the distance from the front; the criterion of the effect of lateral spread becomes not $\eta_s R$ but $\eta_j R$. It follows from the table that for $R = 0.3$ cm when $q_e \lessdot 150$ MW/cm², for $\eta_j R \approx 3$ to 5, cycles should be achieved which are close to a normal luminous detonation; in the region $q_e \leqslant 30$–70 MW/cm², when $\eta_j R \approx 1$ to 2, cycles of incomplete absorption may be achieved, with a marked deviation from a full detonation.

We shall derive the results of the calculations by the quasi-one-dimensional procedure described above for the case $q_a = 100$ MW/cm^2 and R = 3 mm for an air density $\rho_a = 0.316$ of the normal density.

After a time of 2.6 μsec in this variation, the same energy is supplied as in the previous case. Figure 5 shows the shape of the channel for two instants: 1.2 and 2.4 μsec. It can be seen that a long and narrow channel is formed also, and the velocity of the shock front is found to be almost constant (starting from an instant of time of about 0.5 μsec) and equal to approximately 17-18 km/sec, which is considerably lower than it would be in a complete absorption cycle (27 km/sec, see Table 1). The quasisteadiness of motion of the plug of compressed gas immediately behind the front is explained by the origination of a sonic section (the dashes in Fig. 5), which does not transmit a perturbation to the front. The optical thickness of the plug remains constant, which ensures a constant value of the energy absorbed in the plug and expended on maintaining the motion of the front. The plane, where the Jouguet condition is satisfied, lies at a distance of approximately 0.1cm from the shock front, i.e., the thickness of the compressed layer absorbing the radiation is small in comparison with the radius R and in comparison with the distance traveled by the shock wave. We note that in this narrow region there ocurred a reasonable number (about 30) of calculated points, which provided sufficient accuracy for determining the parameters in this zone and the velocity of propagation of the wave D.

Figure 6 gives the distribution of the parameters in the vicinity of the shock front: the pressure p, bar; the ratio of the flux F to the magnitude of the incident flux F_0; the temperature T, eV, at a distance of x, mm. It can be seen that in the section before the Jouguet plane, marked by a dashed line, approximately 50% of the radiation energy was absorbed. However, the parameters at the front and in the Jouguet plane are found to be lower than for $q_e = 50$ MW/cm^2. This is due, obviously, to the fact that although the layer between the front and the Jouguet plane is quite narrow, the lateral expansion of the channel nevertheless is affected slightly.

We note that in the versions with weaker radiation absorption in the compressed gas near the front it is necessary to take account of the effects due to the penetration of the radiation which has remained unabsorbed up to the obstacle and the formation of a vapor jet flowing in the channel.

In conclusion, we note that it would be interesting to investigate the propagation of luminous detonation waves in an incomplete absorption cycle.

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ESTIMATES OF THE NORMAL VELOCITIES OF PROPAGATION OF LAMINAR AND VERY SMALL-SCALED TURBULENT FLAMES

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On the most general assumptions (taking account of the Lewis–Semenov number, thermal expansion, variability of thermophysical parameters, etc.), analytical estimates are obtained for the normal velocities of combustion of laminar and turbulent flames. In the case of an Arrhenius dependence of the reaction velocity on the temperature, the combustion velocity is represented by an asymptotic series with respect to the Frank-Kamenetskii dimensionless temperature; for turbulent flames, with respect to a parameter of the relative scale of turbulence. The final results over a wide range of change of parameters are compared with a numerical calculation on a computer of the exact equations and with the relations obtained by the method of combined asymptotic expansions.

1. Mathematical Formulation of the Problem. Laminar Flame

When the temperature dependence of the rate of the volume heat release is determined by the Arrhenius law

$$\Phi = (\rho(T))^n v^n z(T)\exp(-E/RT), \tag{1.1}$$

the thermal diffusion mechanism of propagation of a one-dimensional steady flame is described [1] by the system of equations

$$\begin{gathered} dp/du = v^n k(u) f(u)/p - \omega; \\ (1/L)dv/du = 1 - \omega(v-u)/p,\ 0 < u < 1 \end{gathered} \tag{1.2}$$

and with the boundary conditions

$$u = 0,\ p = 0,\ v = 0; \tag{1.3}$$

$$u = 1,\ p = 0; \tag{1.4}$$

$$f(u) = \begin{cases} \exp(-\theta_0 u/(1-\sigma u)), & 0 \leqslant u \leqslant \varepsilon \\ 0, & \varepsilon < u \leqslant 1. \end{cases} \tag{1.5}$$

The "cutoff" equation (1.5) of the heat release (ε is the "cutoff" parameter) ensures the existence of an eigenvalue ω_0 of the problem (1.1)-(1.4), which is unique when $1 \leq \mathrm{Le} < \infty$ [1]. The question of uniqueness when $\mathrm{Le} < 1$ still does not have a solution.

The relations between the dimensionless and dimensional quantities are

$$u = (T_+ - T)/(T_+ - T_-);\ p = -(\lambda/\lambda_+)du/d\xi;\ \xi = x/x_+,\ k(u) = (\lambda/\lambda_+)(\rho/\rho_+)^n z/z_+;$$

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